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The Relativistic Effects on the Violation of the Bell's Inequality for Three Qubit W State

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Abstract: In this paper we are going to calculate the correlation function and Bell's inequality for three qubit W state under the Lorentz transformations. This survey is based on the introduction of two different expressions of spin observable were presented by Lee-Young and Kim-Son.

Key words: Entanglement, Spin, Bell's inequality, W state, Qubit

1. INTRODUCTION

The non-local character of quantum mechanics is verified by quantum entanglement. A state is named entangled if it is unfactorizable. In [1] it is found that a decrease in the degree of violation due to the motion of the particle rather than the observer. The entanglement of two particles moving in opposite directions is considered by [2]. They presented that Wigner rotation under Lorentz boost is a local unitary operation and so the entanglement is Lorentz invariant. Ahn et al.([4-11]) studied the Bell observable for entangled states in the rest frame seen by the moving observer presented that the entangled states satisfy Bell's inequality when the boost speed extents the speed of light.
Maximum violation of the Bell's inequality is calculated by Kim in [2] for two particles using the Pauli-Lubanski pseudovector and the generators of the Poincare' group as an observable. In our latest paper [13] we have shown that the maximum violation of Bell's inequality for Greenberger-Horne-Zeilinger (GHZ) state. In this work, we are going to obtain the maximum violation of Bell's inequality for W state in the rest and moving frames then we compare these two maximums.
This paper is organized as follows: In section 1 we first calculate correlation function for three qubit W state in the rest and moving frames in basis of Lee-Young approach [4]. Then we study violation of associated Bell's inequality in chosen directions. Section 2 is devoted to calculation of the correlation function

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and Bell's inequality in Kim-Son approach [2]. In section 3 we enumerate conclusions of this work.

2. THEORY

A. Relativistic Bell's inequality for three qubit W state in Lee-Young approach

The relativistic spin observable for a particle which moves in z direction is given by [6]:

$$\hat{a} = \frac{a_x \sigma_x + a_y \sigma_y + a_z \sigma_z \cosh \xi}{\sqrt{1 + a_z^{\,2} \sinh^2 \xi}}, \tag{1}$$

for three particles which move in z direction, we have three spin observables "$\hat{a}$,$\hat{b}$,$\hat{c}$ " which σ_i 's are the Pauli matrices and $\tanh \xi \equiv \beta_p$ represents the velocity of the particles. The expectation value of joint spin measurement for particles (correlation function) can be expressed as:

$$E(\vec{a},\vec{b},\vec{c}) \equiv \langle \psi | \hat{a} \otimes \hat{b} \otimes \hat{c} | \psi \rangle. \tag{2}$$

For three qubit W state where move in z direction we can write:

$$|\psi\rangle = |W\rangle |\vec{p}_1 \vec{p}_2 \vec{p}_3\rangle, \tag{3}$$

where p_i 's are the momentum of the particles and we know W state is one of the most important three qubit state and

$$|W\rangle = \frac{1}{\sqrt{3}} \left(|110\rangle + |101\rangle + |011\rangle \right). \tag{4}$$

So the correlation function is obtained by:

$$E(\vec{a},\vec{b},\vec{c}) = \langle W | \frac{a_x \sigma_x + a_y \sigma_y + a_z \sigma_z \cosh \xi}{\sqrt{1 + a_z^{\,2} \sinh^2 \xi}} \otimes \frac{b_x \sigma_x + b_y \sigma_y + b_z \sigma_z \cosh \xi}{\sqrt{1 + b_z^{\,2} \sinh^2 \xi}} \otimes \frac{c_x \sigma_x + c_y \sigma_y + c_z \sigma_z \cosh \xi}{\sqrt{1 + c_z^{\,2} \sinh^2 \xi}} |W\rangle, \tag{5}$$

By using the following relations:

$$\sigma_x |0\rangle = |1\rangle \quad \sigma_y |0\rangle = i\,|1\rangle \quad \sigma_z |0\rangle = |0\rangle,$$

$$\sigma_x |1\rangle = |0\rangle \quad \sigma_y |1\rangle = -i\,|0\rangle \quad \sigma_z |1\rangle = -|1\rangle,$$

we can rewrite the correlation function:

$$E\left(\vec{a},\vec{b},\vec{c}\right)$$
$$=\frac{a_z b_z c_z \cosh^3\xi-\frac{2}{3}\cosh\xi\left(a_x b_x c_z+a_x b_z c_x+a_z b_x c_x+a_y b_y c_z+a_y b_z c_y+a_z b_y c_y\right)}{\sqrt{\left(1+a_z{}^2\sinh^2\xi\right)\left(1+b_z{}^2\sinh^2\xi\right)\left(1+c_z{}^2\sinh^2\xi\right)}}. \tag{6}$$

The Bell's inequality to the case of three particles in terms of correlation functions is proposed by [9]:

$$\varepsilon=\left|E\left(\vec{a},\vec{b},\vec{c}'\right)+E\left(\vec{a},\vec{b}',\vec{c}\right)+E\left(\vec{a}',\vec{b},\vec{c}\right)-E\left(\vec{a}',\vec{b}',\vec{c}'\right)\right|\leq 2. \tag{7}$$

For the following vector set:

$$\vec{a}=\vec{b}=\vec{c}=\hat{y}\,,\qquad \vec{a}'=\vec{b}'=\vec{c}'=\hat{z}\,, \tag{8}$$

we have

$$E\left(\vec{a},\vec{b},\vec{c}'\right)=E\left(\vec{a},\vec{b}',\vec{c}\right)=E\left(\vec{a}',\vec{b},\vec{c}\right)=-\frac{2}{3},\ E\left(\vec{a}',\vec{b}',\vec{c}'\right)=1, \tag{9}$$

$$\varepsilon=3. \tag{10}$$

In this case we have the maximum violation of Bell's inequality for W state in non-relativistic case.

Now, we study the effect of Lorentz transformation on Bell's inequality. This effect is considered by two transformations; transformation of the observables and the state distinctly.

I- The effect of Lorentz transformation on the observables

This transformation for one particle is given by [6]:

$$\vec{A}\equiv\vec{a}_R.R(\vec{p}_R)\Rightarrow\vec{A}=\begin{pmatrix} a_x\left(\cos^2\theta-\cosh\eta\sin^2\theta\right)-a_z\left(1+\cosh\eta\right)\sin\theta\cos\theta \\ a_y \\ a_x\left(1+\cosh\eta\right)\sin\theta\cos\theta-a_z\left(\sin^2\theta-\cosh\eta\cos^2\theta\right)\end{pmatrix}, \tag{11}$$

where

$$\tan\theta\equiv\frac{E_p\sinh\chi}{p}=\frac{\sinh\chi}{\tanh\xi}, \tag{12}$$

that χ is the boost speed. Also, we have

$$\sqrt{1+a_z{}^2\sinh^2\xi}\rightarrow\left|\vec{a}_{\Lambda p}\right|=\sqrt{1+\sinh^2\eta(a_x\sin\theta+a_z\cos\theta)^2}, \tag{13}$$

where

$$\tanh\eta \equiv \frac{|\vec{p}_\Lambda|}{E_{\Lambda p}} = \frac{\sqrt{\left(\tanh^2\xi + \sinh^2\chi\right)}}{\cosh\chi}. \quad (14)$$

The same relations existed for b and c observables in (11) and (13).

II- The effect of Lorentz transformation on the state

In this case, we present the following mapping:

$$|\psi\rangle \to |\psi'\rangle = \frac{1}{\sqrt{3}}\Big(w_1|000\rangle + w_2|001\rangle + w_3|010\rangle + w_4|011\rangle$$
$$+ w_5|100\rangle + w_6|101\rangle + w_7|110\rangle + w_8|111\rangle\Big)|\vec{p}_1\vec{p}_2\vec{p}_3\rangle_\Lambda. \quad (15)$$

The Wigner representation of the Lorentz group for spin 1/2 with setting the boost and particle moving directions in the rest frame to be $\hat{\beta} = \hat{x}$ and $\hat{p} = \hat{z}$ respectively is:

$$D\big(W(\Lambda, p_1)\big) = D\big(W(\Lambda, p_2)\big) = D\big(W(\Lambda, p_3)\big)$$
$$= \exp\left(-i\,\frac{\sigma_y}{2}\delta\right) = \begin{pmatrix} \cos\frac{\delta}{2} & -\sin\frac{\delta}{2} \\ \sin\frac{\delta}{2} & \cos\frac{\delta}{2} \end{pmatrix}. \quad (16)$$

by the relations (15) and (16) we obtain:

$$w_1 = 3\sin^2\frac{\delta}{2}\cos\frac{\delta}{2},$$
$$w_2 = w_3 = w_5 = \sin^3\frac{\delta}{2} - 2\sin\frac{\delta}{2}\cos^2\frac{\delta}{2},$$
$$w_4 = w_6 = w_7 = \cos^3\frac{\delta}{2} - 2\sin^2\frac{\delta}{2}\cos\frac{\delta}{2}, \quad (17)$$
$$w_8 = 3\sin\frac{\delta}{2}\cos^2\frac{\delta}{2}.$$

Finally by the use of two sections above the transformed correlation function is:

$$E'\left(\vec{a},\vec{b},\vec{c}\right) = \frac{2}{3\sqrt{\left[1+\sinh^2\eta\left(a_x\sin\theta + a_z\cos\theta\right)^2\right]}}$$
$$\times \frac{1}{\sqrt{\left[1+\sinh^2\eta\left(b_x\sin\theta + b_z\cos\theta\right)^2\right]\left[1+\sinh^2\eta\left(c_x\sin\theta + c_z\cos\theta\right)^2\right]}}$$

$$\times\Big[A_xB_xC_x\left(-3\sin\delta+\frac{9}{2}\sin^3\delta\right)$$
$$+\left(A_xB_xC_z+A_xB_zC_x+A_zB_xC_x\right)\left(-\cos^3\delta+\frac{7}{2}\sin^2\delta\cos\delta\right)$$
$$+\left(A_xB_zC_z+A_zB_xC_z+A_zB_zC_x\right)\left(\frac{7}{2}\sin\delta-\frac{9}{2}\sin^3\delta\right)$$
$$-\sin\delta\left(A_xB_yC_y+A_yB_xC_y+A_yB_yC_x\right)$$
$$-\cos\delta\left(A_zB_yC_y+A_yB_zC_y+A_yB_yC_z\right)$$
$$+\frac{1}{2}A_zB_zC_z\left(3\cos^3\delta-6\sin^2\delta\cos\delta\right)\Big]. \tag{18}$$

Here, we consider two limiting cases:

A. If $\chi\to 0$ then $\theta\to 0$, $\eta\to\xi$, $\delta\to 0$ and we have:

$$E'\left(\vec{a},\vec{b},\vec{c}\right)\to\frac{1}{\sqrt{\left(1+a_z{}^2\sinh^2\xi\right)\left(1+b_z{}^2\sinh^2\xi\right)\left(1+c_z{}^2\sinh^2\xi\right)}}$$
$$\times\left[\begin{array}{l} a_zb_zc_z\cosh^3\xi-\frac{2}{3}\cosh\xi(a_xb_xc_z+a_xb_zc_x+a_zb_xc_x \\ \qquad\qquad\qquad +a_yb_yc_z+a_yb_zc_y+a_zb_yc_y)\end{array}\right]. \tag{19}$$

we find that it is the same as correlation function in (6).

B. If $\xi\to 0$ then $\theta\to\pi/2,\eta\to\chi$, $\delta\to 0$ and we obtain:

$$E'\left(\vec{a},\vec{b},\vec{c}\right)\to\frac{1}{\sqrt{\left(1+a_x{}^2\sinh^2\xi\right)\left(1+b_x{}^2\sinh^2\xi\right)\left(1+c_x{}^2\sinh^2\xi\right)}}$$
$$\times[-a_zb_zc_z+\frac{2}{3}\left(a_yb_yc_z+a_yb_zc_y+a_zb_yc_y\right) \tag{20}$$
$$+\frac{2}{3}\cosh^2\chi\left(a_xb_xc_z+a_xb_zc_x+a_zb_xc_x\right)].$$

In according to (7) for transformed Bell's inequality we have:

$$\varepsilon'=\left|E'\left(\vec{a},\vec{b},\vec{c}'\right)+E'\left(\vec{a},\vec{b}',\vec{c}\right)+E'\left(\vec{a}',\vec{b},\vec{c}\right)-E'\left(\vec{a}',\vec{b}',\vec{c}'\right)\right|\le 2. \tag{21}$$

Using vectors (8) in (18) we have

$$E'\left(\vec{a},\vec{b},\vec{c}'\right)=E'\left(\vec{a},\vec{b}',\vec{c}\right)=E'\left(\vec{a}',\vec{b},\vec{c}\right)$$
$$=\frac{2}{3\sqrt{1+\sinh^2\eta\cos^2\theta}}[\sin\delta\left(1+\cosh\eta\right)\sin\theta\cos\theta \qquad (22)$$
$$+\cos\delta\left(\sin^2\theta-\cosh\eta\cos^2\theta\right)],$$

and

$$E'\left(\vec{a}',\vec{b}',\vec{c}'\right)=\frac{2}{3\sqrt{\left(1+\sinh^2\eta\cos^2\theta\right)^3}}$$
$$\times\left[\left(1+\cosh\eta\right)^3\sin^3\theta\cos^3\theta\left(3\sin\delta-\frac{9}{2}\sin^3\delta\right)\right.$$
$$+\left(1+\cosh\eta\right)^2\sin^2\theta\cos^2\theta\left(\sin^2\theta-\cosh\eta\cos^2\theta\right)\left(3\cos^3\delta-\frac{21}{2}\sin^2\delta\cos\delta\right)$$
$$+\left(1+\cosh\eta\right)\sin\theta\cos\theta\left(\sin^2\theta-\cosh\eta\cos^2\theta\right)^2\left(\frac{27}{2}\sin^3\delta-\frac{21}{2}\sin\delta\right)$$
$$\left.+\frac{1}{2}\left(\sin^2\theta-\cosh\eta\cos^2\theta\right)^3\left(6\sin^2\delta\cos\delta-3\cos^3\delta\right)\right], \qquad (23)$$

in this case we obtain the Bell's inequality as follows:

$$\varepsilon'=\left|\frac{2}{\sqrt{1+\sinh^2\eta\cos^2\theta}}\right.$$
$$\times\left[\sin\delta\left(1+\cosh\eta\right)\sin\theta\cos\theta+\cos\delta\left(\sin^2\theta-\cosh\eta\cos^2\theta\right)\right]$$
$$-\frac{2}{3\sqrt{(1+\sinh^2\eta\cos^2\theta)^3}}\left[\left(1+\cosh\eta\right)^3\sin^3\theta\cos^3\theta\left(3\sin\delta-\frac{9}{2}\sin^3\delta\right)\right.$$
$$+\left(1+\cosh\eta\right)^2\sin^2\theta\cos^2\theta\left(\sin^2\theta-\cosh\eta\cos^2\theta\right)\left(3\cos^2\delta-\frac{21}{2}\sin^2\delta\cos\delta\right)$$
$$+\left(1+\cosh\eta\right)\sin\theta\cos\theta\left(\sin^2\theta-\cosh\eta\cos^2\theta\right)^2\left(\frac{27}{2}\sin^3\delta-\frac{21}{2}\sin\delta\right)$$
$$\left.\left.+\frac{1}{2}\left(\sin^2\theta-\cosh\eta\cos^2\theta\right)^3\left(6\sin^2\delta\cos\delta-3\cos^3\delta\right)\right]\right|.$$
$$(24)$$

Now, two limiting cases of the above relation as $\chi \to 0$ and $\xi \to 0$ become

$$\varepsilon' \to 0, \tag{25}$$

where demonstrated that vectors (8) maximally violates Bell's inequality in this case.

The relativistic correlation function is not invariant in this case nevertheless in next section for another observable this function will be invariant in relativistic case.

B. Relativistic Bell's inequality for three qubit W state in Kim-Son approach

Here (similar to previous section) three particles considered to have been +z direction. In the rest frame, the spin observable in the direction $\vec{a}$ is $\vec{a}.\vec{S}$,where $\vec{S} = (\hbar/2)\vec{\sigma}$ is the spin operator. Using the Pauli-Lubanski pseudovector with the generators of the Poincare group [10], the invariant expression measured by the four-dimensional axis is [11]:

$$\hat{O}(a) = \frac{2a^{\mu}W_{\mu}}{m\,c\,\hbar}. \tag{26}$$

The spin vector and the axis should be transformed by the appropriate transformation law. For the observable $\widehat{O}(a,b,c) = \widehat{O}(a) \otimes \widehat{O}(b) \otimes \widehat{O}(c)$ the transformed expectation value $\langle W' | \hat{O}'(a,b,c) | W' \rangle$ can be calculated with regard to the observable transformation as:

$$\begin{aligned}\tilde{O}'(a,b,c) &= U(\Lambda)\tilde{O}(a,b,c)U^{-1}(\Lambda)\\ &= \left(2/mc\hbar\right)^3 a^{\mu}b^{\nu}c^{\rho}U(\Lambda)W_{\mu} \otimes W_{\nu} \otimes W_{\rho}U^{-1}(\Lambda)\\ &= 8\vec{a}\cdot\vec{S}_R \otimes \vec{b}\cdot\vec{S}_R \otimes \vec{c}\cdot\vec{S}_R\,,\end{aligned} \tag{27}$$

where $\vec{S}_R = D(W)\vec{S}D^{-1}(W)$. Using the equation (17), the transformation of the spin can rewritten by the following relation [11]:

$$\begin{aligned}2\vec{a}\cdot\vec{S}_R &= \vec{a}\cdot D(W)\vec{\sigma}D^{-1}(W)\\ &= \begin{pmatrix} a_z \cos\delta - a_x \sin\delta & a_z \sin\delta + a_x \cos\delta - ia_y \\ a_z \sin\delta + a_x \cos\delta + ia_y & -a_z \cos\delta + a_x \sin\delta \end{pmatrix}\\ &= 2\vec{a}_R \cdot \vec{S}.\end{aligned} \tag{28}$$

The same expression are used for $\vec{b}$ and $\vec{c}$, so:

$$\tilde{O}'(a,b,c) = 8\vec{a}_R\cdot\vec{S} \otimes \vec{b}_R\cdot\vec{S} \otimes \vec{c}_R\cdot\vec{S} = \tilde{O}'\left(\vec{a}_R, \vec{b}_R, \vec{c}_R\right), \tag{29}$$

then, the unit vector $\vec{a}$ is transformed as:

$$\vec{a}_R = \left(a_x \cos\delta + a_z \sin\delta, a_y, -a_x \sin\delta + a_z \cos\delta\right), \tag{30}$$

also for $\vec{b}_R$ and $\vec{c}_R$ with b and c as a substitute of a .

After some mathematical manipulations, we have

$$E'\left(\vec{a},\vec{b},\vec{c}\right) = a_z b_z c_z - \frac{2}{3}(a_x b_x c_z + a_x b_z c_x + a_z b_x c_x + a_z b_y c_y + a_y b_z c_y + a_y b_y c_z). \tag{31}$$

Note that the expectation value of W state is invariant under the Lorentz boost. Using the above expectation value and vector set in (8), it can be shown that the violation of Bell's inequality for W state is maintained at any reference frame and we have

$$\varepsilon'\left(\vec{a},\vec{b},\vec{c};W'\right) = \varepsilon\left(\vec{a},\vec{b},\vec{c};W\right) = 3. \tag{32}$$

3. CONCLUSION

In conclusion using Bell’s inequality, we studied the nonlocal quantum properties of W state in relativistic formalism. We first evaluate correlation function for three qubit W state in the rest and moving systems in basis of Lee-Young approach. Then we study violation of associated Bell's inequality in chosen directions and we have calculated correlation function and Bell's inequality in Kim-Son approach. Bell’s inequality is maximally violated in rest frame or in moving frame with rest particles. For W state we presented that in limiting cases Bell’s inequality is maximally violated. Finally interestingly we found that the maximum degree of violation of Bell's inequality for GHZ state is more than W state.

APPENDIX

For proving the relation (6) we have

$$E(\vec{a},\vec{b},\vec{c}) = \langle W| \Big(\frac{a_x\sigma_x + a_y\sigma_y + a_z\sigma_z\cosh\xi}{\sqrt{1+a_z{}^2\sinh^2\xi}} \otimes \frac{b_x\sigma_x + b_y\sigma_y + b_z\sigma_z\cosh\xi}{\sqrt{1+b_z{}^2\sinh^2\xi}} \otimes \frac{c_x\sigma_x + c_y\sigma_y + c_z\sigma_z\cosh\xi}{\sqrt{1+c_z{}^2\sinh^2\xi}}\Big)|W\rangle$$

$$= \frac{1}{3}\left(\langle 110| + \langle 101| + \langle 011|\right)\Big(\frac{a_x\sigma_x + a_y\sigma_y + a_z\sigma_z\cosh\xi}{\sqrt{1+a_z{}^2\sinh^2\xi}} \otimes \frac{b_x\sigma_x + b_y\sigma_y + b_z\sigma_z\cosh\xi}{\sqrt{1+b_z{}^2\sinh^2\xi}} \otimes \frac{c_x\sigma_x + c_y\sigma_y + c_z\sigma_z\cosh\xi}{\sqrt{1+c_z{}^2\sinh^2\xi}}\Big)$$

$$\times\left(\left|110\right\rangle+\left|101\right\rangle+\left|011\right\rangle\right)$$

$$=\frac{a_z b_z c_z \cosh^3\xi-\frac{2}{3}\cosh\xi\left(a_x b_x c_z+a_x b_z c_x+a_z b_x c_x+a_y b_y c_z+a_y b_z c_y+a_z b_y c_y\right)}{\sqrt{\left(1+a_z{}^2\sinh^2\xi\right)\left(1+b_z{}^2\sinh^2\xi\right)\left(1+c_z{}^2\sinh^2\xi\right)}}.$$

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