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Casimir effects of nano objects in fluctuating scalar and electromagnetic fields: Thermodynamic investigating

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Abstract: Casimir entropy is an important aspect of casimir effect and at the nanoscale is visible. In this paper, we employ the path integral method to obtain a general relation for casimir entropy and internal energy of arbitrary shaped objects in the presence of two, three and four dimension scalar fields and the electromagnetic field. For this purpose, using Lagrangian and based on a perturbative approach, a series expansion in susceptibility function of the medium was obtained for the Casimir force between arbitrary shaped objects foliated in a scalar or vector fluctuating field in arbitrary dimensions. The finite temperature corrections are derived and using it, we obtain the casimir entropy and internal energy of two nano ribbons immersed in the scalar field and two nanospheres immersed in the scalar field and the electromagnetic field. The casimir entropy of two nanospheres immersed in the electromagnetic field behave differently in small interval of temperature variations. .

Key words: Casimir entropy, Internal energy, Path integral method, Negative entropy, nano sphere.

1. INTRODUCTION

Since the zero point energy of vacuum fluctuations plays a very important role in the theory of quantized fields, it has boob introduced into various branches of physics [1].

The casimir effect occurs due to the boundary conditions of medium surfaces imposed on the fluctuating field. In the other words, the casimir energy refers to the difference between the energy of the fluctuating field when objectsare present and when the objects are removed to infinity [2,3], therefore, the casimir entropy has been extensively studied [4-7].

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Recently, a group of scientists presented a multi-scattering formalism to study fluctuating fields and casimir effects [8-10]. They used multi-scattering formalism for bodies when they were weakly coupled to the quantum fields [11-13].
In 2012, we used path integral techniques to calculate the casimir force of arbitrary shaped objects immersed in a fluctuating field. It was shown that, in the first order approximation in our method might, the casimir energy be equivalent to weak coupling limit in the multiple-scattering formalism [14].
Recently, scientists have found that the intervals of negative entropy also occur for some geometries [15-18]. Since the specific heat of system is proportional to the variation of the system entropy, where coupling between system and reservoir can be negligible, [5,19-20], the specific heat of the system is defined by the difference between the specific heat of the system as well as reservoir and specific heat of the reservoir. Therefore, in cases where two parallel plates [17] and a sphere beside a plate are analyzed according to Drude model [18], casimir entropy becomes negative.
In this paper, we investigate casimir entropy, using the path integral methods for arbitrary shaped objects which are immersed in a massless scalar field and an electromagnetic field [14].
First, we obtain a general relation for Casimir entropy and internal energy of the system. Then, we apply this procedure to objects immersed in (1+1)D, (2+1)D, (3+1)D scalar fields, and the electromagnetic field.
This paper is organized as follows: in section II, we briefly describe the free energy and entropy based on path integral methods; in section III, we calculate them for 1+1 Dimension, 2+1 Dimension and 3+1 Dimension space-time in the scalar field; Finally in section IV, we obtain free energy and entropy in the electromagnetic field.

2. Free energy and entropy

The casimir free energy of a system in a quantum field theory at finite temperature in general form becomes as follows [appendix A]

$$E = -k_B T \sum_{l=0}^{\infty} tr \ln[G(i\upsilon_l; x, x')] \tag{1}$$

where, $G(i\upsilon_l; x, x')$ is total Green's function of interacting system which defines

$$G(x-x',\omega)=G^0(x-x',\omega)+\int_\Omega d^n z_1 G^0(x-z_1,\omega)[\omega^2\tilde{\chi}(\omega,z_1)]G^0(z_1-x',\omega)+\int_\Omega d^n z_1\int_\Omega d^n z_2 G^0(x-z_1,\omega)[\omega^2\tilde{\chi}(\omega,z_1)]G^0(z_1-z_2,\omega)[\omega^2\tilde{\chi}(\omega,z_2)]G^0(z_2-x',\omega)+... \quad (2)$$

where $\tilde{\chi}(\omega,x)$ is the susceptibility function of the medium with frequency variable and $G^0(x_1-x_2,i\upsilon_l)$ is Green's function of the medium.

In terms of the susceptibility function, the expansion of free energy is achieved by

$$E=k_BT\sum_{l=0}^{\infty}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\int_\Omega d^n x_1...\,d^n x_n G^0(x_1-x_2,i\upsilon_l)...G^0(x_n-x_1,i\upsilon_l)\times\tilde{\chi}(i\upsilon_l,x_1)...\tilde{\chi}(i\upsilon_l,x_n) \quad (3)$$

The casimir entropy is obtained from the casimir free energy, using thermodyna mic relation

$$S=-\frac{\partial E}{\partial T} \quad (4)$$

According to the Eqs.(1) and (4), in terms of total Green's function, casimir entropy of the system is

$$S=k_B\sum_{l=0}^{\infty}tr[\ln G(i\upsilon_l;x,x')+TG^{-1}(i\upsilon_l;x,x')\frac{d}{dT}G(i\upsilon_l;x,x')] \quad (5)$$

According to above relation, in terms of free energy, entropy can be rewritten in the following form

$$S=-\frac{E}{T}-T\frac{d}{dT}(\frac{E}{T}) \quad (6)$$

And internal energy of the system becomes

$$U=-T^2\frac{d}{dT}(\frac{E}{T}) \quad (7)$$

3. Scalar field

A.1+1 Dimension

In this part, we obtain the force induced, casimir entropy, and internal energy of the system which consist of two one dimension objects with susceptibilities $\chi_1(\omega)$ $\chi_2(\omega)$ in the presence of fluctuating massless scalar field. In this case, the fluctuating field is defined in (1+1)-dimensional space-time x=(x, t). The Green's function of the system is given by

$$G^0(\omega; x-x') = \frac{e^{-i\omega|x-x'|}}{2\omega}, \tag{8}$$

Using Eqs.(8) and (3), the free energy of 1+1D in the first approximation becomes

$$E = -k_B T \sum_{l=1}^{\infty} \int dx \int dx' \frac{e^{-2\alpha_l|x-x'|}}{(2\alpha_l)^2} \chi_1(i\alpha_l, x) \chi_2(i\alpha_l, x') \tag{9}$$

Where $\alpha_l = \frac{\upsilon_l}{c} = \frac{2\pi l k_B}{\hbar c}$.

We assume that susceptibility is position independent. The matter distribution is homogeneous, so the dielectric function is defined by

$$\frac{\varepsilon(\omega, x)}{\varepsilon_0} = \begin{cases} \frac{\varepsilon_1(\omega)}{\varepsilon_0}, & a < x < b \\ 1, & b < x < c \\ \frac{\varepsilon_2(\omega)}{\varepsilon_0}, & c < x < d \end{cases} \tag{10}$$

The free energy at finite temperature is given by

$$E = -k_B T \sum_{l=1}^{\infty} \frac{1}{(2\alpha_l T)^2} [\chi_1^2 \frac{b-a}{\alpha_l T} + \chi_2^2 \frac{d-c}{\alpha_l T} - \frac{2\chi_1\chi_2}{(2\alpha_l T)^2} (e^{-2\alpha_l T d} - e^{-2\alpha_l T c})(e^{2\alpha_l T b} - e^{2\alpha_l T a})] \tag{11}$$

which consists of self-energies and interaction energies. By defining

$$a\_b = 2r_1,\ b-a = 2r'',\ c+d = 2r_2,\ d-c = 2r',\ r = r_2 - r_1; \tag{12}$$

the free energy is obtained in terms of distances

$$E = -k_B T \sum_{l=1}^{\infty} \frac{1}{(2\alpha_l T)^2} [\chi_1^2 \frac{r''}{\alpha_l T} + \chi_2^2 \frac{r'}{\alpha_l T} - \frac{2\chi_1\chi_2}{(2\alpha_l T)^2} e^{-2\alpha_l T r} (e^{-2\alpha_l T(r'+r'')} - e^{2\alpha_l T(r'+r'')} + e^{2\alpha_l T(r'-r'')} - e^{-2\alpha_l T(r'-r'')})] \tag{13}$$

According to Eq.(4), the entropy of the system becomes

$$S=-k_B[-\chi_1^2 r''\frac{Zeta[3]}{(\gamma T)^3}-\chi_2^2 r'\frac{Zeta[3]}{(\gamma T)^3}$$
$$+\frac{\chi_1\chi_2}{2(\gamma T)^2}(Li_2(e^{-2\gamma T(r+r'-r'')})-Li_2(e^{-2\gamma T(r+r'+r'')})$$
$$-Li_2(e^{-2\gamma T(r-r'-r'')})+Li_2(e^{-2\gamma T(r-r'+r'')}))$$
$$-\frac{\chi_1\chi_2}{\gamma T}((r+r'-r'')\log(1-e^{-2\gamma T(r+r'-r'')}) \tag{14}$$
$$-(r+r'+r'')\log(1-e^{-2\gamma T(r+r'-r'')})$$
$$-(r-r'-r'')\log(1-e^{-2\gamma T(r-r'-r'')})$$
$$+(r-r'+r'')\log(1-e^{-2\gamma T(r-r'-r'')}))]$$

where $\gamma=\frac{2\pi k_B}{\hbar c}$ and $Li_s(z)$ are polylogarithm functions which are defined by the infinite sum $Li_s(z)=\sum_{k=1}^{\infty}\frac{z^k}{k^s}$. In terms of temperature, the variance of entropy is shown in Fig. 1 and in Fig. 2.

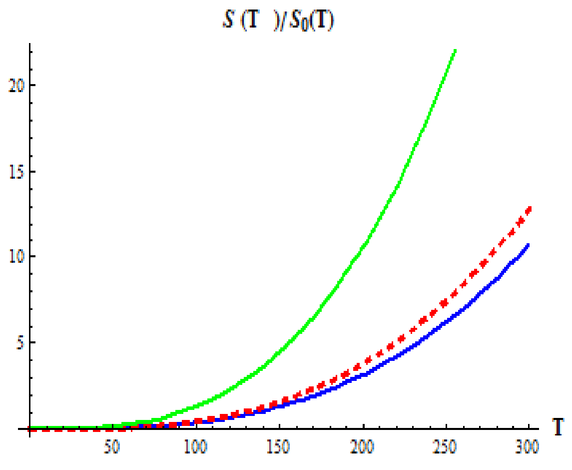

Fig. 1 Entropy of two nano ribbons in one dimension with the same susceptibility in terms of temperature, blue(b-a=2nm,c-b=8nm, d-c=4nm), red(b-a=2nm,c-b=8nm,d-c=8nm),green(b-a=10nm,c-b=8nm,d-c=8nm).

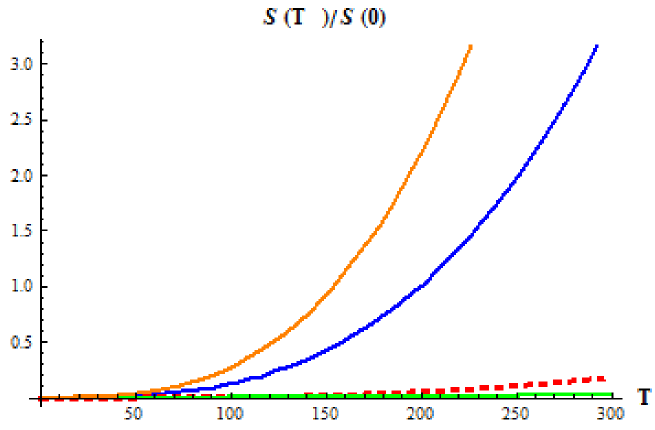

Fig. 2 Entropy of two nano ribbons in one dimension with (b-a=1nm, c-b=4nm, d-c=1nm) and susceptibilities blue (χ1= 11.68 and χ= 2.6), red(χ1= 11.68 and χ2= 1000), green(χ1= 11.68 and χ2= 6000), orange(χ1= 2 and χ2= 3) in terms of temperature.

The force induced resulting from the fluctuating massless scalar field on two objects is given by

$$F=\frac{\partial E}{\partial r}=-k_B\frac{\chi_1\chi_2}{2\gamma}(-\log[1-e^{-2\gamma T(r+r'+r")}]+\log[1-e^{-2\gamma T(r-r'+r")}] \\ +\log[1-e^{-2\gamma T(r+r'-r")}]-\log[1-e^{-2\gamma T(r-r'-r")}]) \quad (15)$$

And internal energy is

$$U=k_B\left[\frac{3(\chi_1^2 r"+\chi_2^2 r')}{4\gamma^3T^2}+\frac{\chi_1\chi_2}{2\gamma^4T^3}[Li_4(e^{-2\gamma T(r+r'+r")}) \right. \\ -Li_4(e^{-2\gamma T(r-r'-r")})+Li_4(e^{-2\gamma T(r-r'+r")}) \\ -Li_4(e^{-2\gamma T(r+r'-r")})] \\ -\frac{\chi_1\chi_2}{4\gamma^3T^2}[-(r+r'+r")Li_3(e^{-2\gamma T(r+r'+r")}) \\ +(-r+r'+r")Li_3(e^{-2\gamma T(r-r'-r")}) \\ +(-r+r'-r")Li_3(e^{-2\gamma T(r-r'+r")}) \\ -(r+r'-r")Li_3(e^{-2\gamma T(r+r'-r")})]]. \quad (16)$$

B. 2+1 dimension space-time

The behavior of two objects in the presence of the fluctuating massless scalar field, which is defined in 1+2 dimensional space- time, is investigated. Therefore, the green's function of the system become

$$G_0(\omega, x-x') = \frac{i\hbar}{2\pi} K_0(i\omega|x-x'|) \tag{17}$$

in which $K_0(i\omega|x-x'|)$ is the modified Bessel function of the second kind. Given Eq.(3), in the first approximation, the free energy is obtained by

$$E = -\frac{k_B T}{4\pi^2}\sum_{l=1}^{\infty}\int d^2x \int d^2x' K_0^2(\alpha_l |x-x'|)\chi_1(i\alpha_l, x)\chi_2(i\alpha_l, x'), \tag{18}$$

where self energies are ignored. In objects whose susceptibilities are frequency independent, the casimir entropy of the system becomes

$$S = -\frac{k_B}{4\pi^2}\sum_{l}\int d^2x \int d^2x' \chi_1(x)\chi_2(x')[K_0^2(\gamma l T|x-x'|) \\ -2\gamma l T|x-x'|K_0(\gamma l T|x-x'|)K_1(\gamma l T|x-x'|)] \tag{19}$$

and the internal energy of the system is given by

$$U = -\frac{k_B}{2\pi^2}\sum_{l}\int d^2x \int d^2x' \chi_1(x)\chi_2(x')\, \gamma l T^2 \\ |x-x'|K_0(\gamma l T|x-x'|)K_1(\gamma l T|x-x'|) \tag{20}$$

C. 3+1-dimensional space-time

In this section, we restrict ourselves to (3+1)-dimensional space-time x=(x, t). In this case, the Green's function of the system becomes

$$G_0(\omega, x-x') = \frac{1}{4\pi}\frac{e^{-i\omega|x-x'|}}{|x-x'|}, \tag{21}$$

and the free energy in the first approximation is given by

$$E = -\frac{k_B T}{16\pi}\sum_{l=1}^{\infty}\int d^3x \int d^3x' \frac{e^{-2\upsilon_l|x-x'|}\chi_1(x,\upsilon_l)\chi_2(x',\upsilon_l)}{|x-x'|^2}, \tag{22}$$

When susceptibilities are independent of frequency, we have

$$E = -\frac{k_B T}{16\pi}\int d^3x \int d^3x' \frac{\chi_1(x)\chi_2(x')}{|x-x'|^2}\left(\frac{1}{1-e^{-2\gamma T|x-x'|}}\right), \tag{23}$$

The system's internal energy and entropy are achieved by

$$U = \frac{k_B}{8\pi}\int d^3x \int d^3x'\, \gamma T^2 \frac{\chi_1(x)\chi_2(x')}{|x-x'|}\frac{e^{-2\gamma T|x-x'|}}{(1-e^{-2\gamma T|x-x'|})^2}, \tag{24}$$

$$S=-\frac{k_B}{16\pi}\int d^3x\int d^3x'\,\frac{\chi_1(x)\chi_2(x')}{|x-x'|^2}[\frac{2\gamma T\,|x-x'|e^{-2\gamma T|x-x'|}}{(1-e^{-2\gamma T|x-x'|})^2}+\frac{1}{1-e^{-2\gamma T|x-x'|}}], \qquad (٢٥)$$

In the lower temperature, we can use the expansion form of the previous relation to obtain

$$S=-\frac{k_B}{16\pi}\int d^3x\int d^3x'\,\frac{\chi_1(x)\chi_2(x')}{|x-x'|^2}[\frac{1}{\gamma T\,|x-x'|}+\frac{1}{2}$$
$$+\frac{(\gamma T)^3}{45}|x-x'|-\frac{4(\gamma T)^5}{945}|x-x'|^3+...], \qquad (26)$$

We consider two spheres, with radius a and b. The distance between their centers is R>a+b . The susceptibilities of the spheres are $\chi_1(x)=\chi_1\delta(r-a)$ and $\chi_2(x)=\chi_2\delta(r-b)$, where r and r' are radial coordinates in spherical coordinate systems, and R lies along the z axis of both coordinate systems. The distance between points on the spheres is

$$|x-x'|=\sqrt{R^2+a^2+b^2-2ab\cos\gamma-2R(a\cos\theta-b\cos\theta)},$$
$$\cos\gamma=\cos\theta\cos\theta'+\sin\theta\sin\theta'\cos(\phi-\phi') \qquad (27)$$

According to Eq.(26), we have

$$S=-\frac{k_B}{16\pi}\int d\Omega\int d\Omega'\,\chi_1\chi_2[\frac{1}{\gamma T\,|x-x'|^3}+\frac{1}{2|x-x'|^2}$$
$$+\frac{(\gamma T)^3}{45}|x-x'|-\frac{4(\gamma T)^5}{45}|x-x'|^3+...], \qquad (28)$$

by using

$$\int d\Omega\int d\Omega'|x-x'|^p=(4\pi)^2R^pP_p(\hat{a},\hat{b})$$
$$P_{-1}=1,$$
$$P_{-2}=\frac{1}{4\hat{a}\hat{b}}(\ln[\frac{1-(\hat{a}+\hat{b})^2}{1-(\hat{a}-\hat{b})^2}]+\hat{a}\ln[\frac{(\hat{b}+1)^2-\hat{a}^2}{(\hat{b}-1)^2-\hat{a}^2}]$$
$$+\hat{b}\ln[\frac{(\hat{a}+1)^2-b^2}{(\hat{a}-1)^2-\hat{b}^2}]),$$
$$P_{-3}=-\frac{1}{4\hat{a}\hat{b}}\ln[\frac{1-(\hat{a}+\hat{b})^2}{1-(\hat{a}-\hat{b})^2}], \qquad (29)$$
$$P_p(\hat{a},\hat{b})=\frac{1}{4\hat{a}\hat{b}}\frac{1}{(p+2)(p+3)}[(1+\hat{a}+\hat{b})^{p+3}+(1-\hat{a}-\hat{b})^{p+3}$$
$$-(1-\hat{a}+\hat{b})^{p+3}-(1+\hat{a}-\hat{b})^{p+3}]$$
$$p=0,1,2,3,...$$

Where $\hat{a}=\frac{a}{R}$ and $\hat{b}=\frac{b}{R}$ and $P_{p-1}=\frac{R^{-p}}{1+p}\frac{\partial}{\partial R}R^{p+1}P_p(\hat{a},\hat{b})$, the casimir entropy of this system is obtained by

$$S=-\frac{k_B}{16\pi}\chi_1\chi_2\left[\frac{1}{\gamma T}(4\pi)^2R^{-3}P_{-3}(\hat{a},\hat{b})+\frac{1}{2}(4\pi)^2R^{-2}P_{-2}(\hat{a},\hat{b})\right.$$
$$\left.+\frac{(\gamma T)^3}{45}(4\pi)^2RP_1(\hat{a},\hat{b})-\frac{4}{945}(\gamma T)^5(4\pi)^2R^3P_3(\hat{a},\hat{b})+...\right] \qquad (30)$$

The casimir entropy becomes positive in all temperatures Fig. 3 .

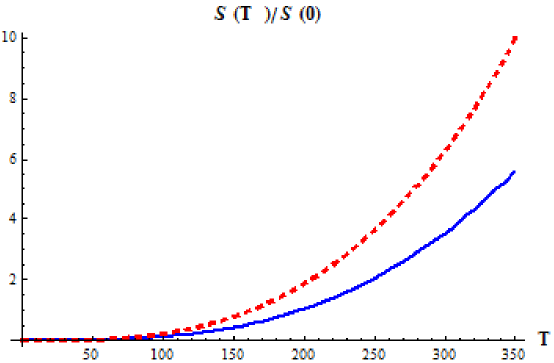

Fig. 3 Entropy of the two spheres of radius a and b and distance between their centers R, immersed in scalar 3+1 dimension field in terms of temperature, blue(a = 1nm,b = 2nm,R = 10nm), red(a = 1nm,b = 2nm,R = 20nm) and susceptibilities (χ1= 11.68 and χ2= 2.6).

4. Electromagnetic field

We use the previous approach in the previous section to find the partition function in terms of the susceptibility of the medium as follows

$$E=k_BT\sum_{l=0}^{\infty}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\int_{\Omega}d^nx_1...\,d^nx_nG^0_{i_1i_2}(x_1-x_2,i\upsilon_l)...G^0_{i_ni_1}(x_n-x_1,i\upsilon_l)$$
$$\tilde{\chi}(i\upsilon_l,x_1)...\tilde{\chi}(i\upsilon_l,x_n) \qquad (31)$$

where the free Green's function of the electromagnetic field is $G^0_{ij}(x_i-x_j,i\upsilon_l)$. We introduce $\vec{r}=x-x'$ and find

$$G^0_{ij}(\vec{r}, i\upsilon_l) = \frac{\upsilon_l^2}{c^2}\frac{e^{-\frac{\upsilon_l r}{c}}}{4\pi r}$$
$$[\delta_{ij}(1+\frac{c}{\upsilon_l r}+\frac{c^2}{\upsilon_l^2 r^2}) - \frac{r_i r_j}{r^2}(1+\frac{3c}{\upsilon_l r}+\frac{3c^2}{\upsilon_l^2 r^2})] + \frac{1}{3}\delta_{ij}\delta^3(\vec{r}) \quad (32)$$

We calculate the interaction energy, casimir entropy, and internal energy of a system which are composed of two dielectrics with volumes V_1 and V_2 and susceptibilities χ_1 and χ_2, respectively. The first relevant nonzero term corresponds to n=2, which is given by

$$E = -\frac{1}{2}k_B T\sum_{l=1}^{\infty}\int d^3x\int d^3x' G^0_{ij}(x-x', i\upsilon_l)G^0_{ji}(x'-x, i\upsilon_l)\chi_1(i\upsilon_l, x)\chi_2(i\upsilon_l, x') \quad (33)$$

Substituting the Green's function (32) for (33), we find

$$E = -k_B T\sum_{l=1}^{\infty}\int d^3x\int d^3x' \chi_1(i\upsilon_l, x)\chi_2(i\upsilon_l, x')h(\upsilon_l, |x-x'|) \quad (34)$$

Where

$$h(\upsilon_l, |x-x'|) = \frac{e^{-\frac{2\upsilon_l}{c}|x-x'|}}{8\pi^2}\{\frac{(\frac{\upsilon_l}{c})^4}{|x-x'|^2} + \frac{2(\frac{\upsilon_l}{c})^3}{|x-x'|^3} + \frac{5(\frac{\upsilon_l}{c})^2}{|x-x'|^4} + \frac{6\frac{\upsilon_l}{c}}{|x-x'|^5} + \frac{3}{|x-x'|^6}\} \quad (35)$$

Whenever the susceptibilities are independent of frequency, we do the summation over l and use the expansion of exponential term, the free energy, internal energy and entropy of the system, respectively, are obtained by

$$E = -k_B T\chi_1\chi_2\int d^3x\int d^3x'[\frac{55}{\gamma T|x-x'|^7} + \frac{3}{2|x-x'|^6}$$
$$-\frac{\gamma T}{4|x-x'|^5} - \frac{\gamma^3 T^3}{240|x-x'|^3} + ...] \quad (36)$$

$$U = k_B\chi_1\chi_2\int d^3x\int d^3x'[\frac{55}{\gamma|x-x'|^7} + \frac{\gamma}{4|x-x'|^5}T^2 +$$
$$\frac{\gamma^3 T^4}{80|x-x'|^3} - \frac{73\gamma^5 T^6}{6048|x-x'|} + ...] \quad (37)$$

$$S = -k_B\chi_1\chi_2\int d^3x\int d^3x'[\frac{3}{2|x-x'|^6} - \frac{\gamma}{2|x-x'|^5}T$$
$$-\frac{\gamma^3 T^3}{60|x-x'|^3} + \frac{73\gamma^5 T^5}{5040|x-x'|} + ...] \quad (38)$$

For two spheres of radii a and b, the distance between their centers R>a+b, with susceptibilities $\chi_1(x)=\chi_1\delta(r-a)$ and $\chi_2(x)=\chi_2\delta(r-b)$, where r and r' are radial coordinates in spherical coordinate systems, Casimir entropy in natural unit becomes

$$S=-(4\pi)^2\chi_1\chi_2[\frac{3}{2}R^{-6}P_{-6}(\hat{a},\hat{b})-\pi R^{-5}P_{-5}(\hat{a},\hat{b})T \\ -\frac{2\pi^3}{15}R^{-3}P_{-3}(\hat{a},\hat{b})T^3+\frac{2\pi^5}{315}R^{-1}P_{-1}(\hat{a},\hat{b})T^5+...] \quad (39)$$

Casimir entropy of the system related to temperature is shown in fig.4, in which entropy becomes negative in small interval of temperature variations.

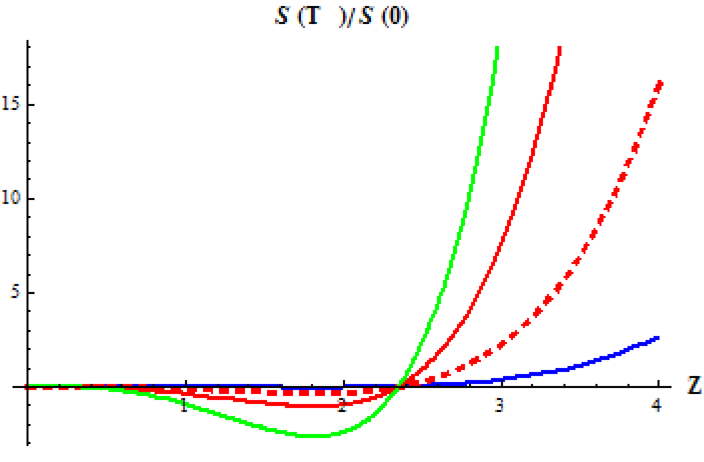

Fig. 4 Entropy of the two spheres, of radius a and b and distance between their centers R(a = 1nm,b = 2nm,R = 10mm), immersed in electromagnetic field in terms of Z = 4πRT ,with susceptibilities, blue(χ1χ2= 1), red dash (χ1χ2= 6), red(χ1χ2= 20), green(χ1χ2= 50)

5. Conclusion

In this paper, casimir entropy and internal energy of arbitrary shaped objects im mersed in the fluctuating massless scalar field and the electromagnetic field are calculated by path integral methodes. The casimir entropy of the objects which are immersed in the massless scalar field in all temperatures becomes positive. However, the value of entropy observed in small interval of temperature variations of two nanospheres in the presence of electromagnetic field is negative. The result is in agreement with multiple scattering methods.

Appendices

A. Massless Scalar field

The Lagrangian of total system consists of a massless scalar field in N+1-dimensional space-time x=(x,t). Lagrangian of the scalar field, medium and interaction between scalar field and medium, respectively, are given by

$$L=\frac{1}{2}\partial_{\mu}\varphi(x)\partial^{\mu}\varphi(x)+\frac{1}{2}\int_{0}^{\infty}d\omega\left(\dot{Y}_{\omega}^{2}(x)-\omega^{2}Y_{\omega}^{2}(x)\right)+\int_{0}^{\infty}d\omega f(x,\omega)\dot{Y}_{\omega}(x)\varphi(x) \qquad (1\text{-}1)$$

where $f(x,\omega)$ is the coupling of the scalar field with its medium. When we have the total Lagrangian, we can quantize the total system using path-integral techniques Generating functional is an important quantity in any field theory with n-point correlation functions, which is obtained by taking successive functional derivatives. Here, two-point correlation functions in terms of the susceptibility of the medium is calculated [21,22]. The interacting generating-function becomes

$$\begin{aligned}W[J,J_{\omega}]&=e^{\frac{i}{\hbar}\int d^{n}x\int dt\int_{0}^{\infty}d\omega f(x,\omega)(\frac{\hbar}{i}\frac{\delta}{\delta J(x)})\frac{\partial}{\partial t}(\frac{\hbar}{i}\frac{\delta}{\delta J_{\omega}(x)})}W_{0}[J,J_{\omega}]\\&=e^{\frac{i}{\hbar}\int d^{n}x\int dt\int_{0}^{\infty}d\omega f(x,\omega)(\frac{\hbar}{i}\frac{\delta}{\delta J(x)})\frac{\partial}{\partial t}(\frac{\hbar}{i}\frac{\delta}{\delta J_{\omega}(x)})}\\&\times e^{-\frac{1}{2\hbar^{2}}\int d^{n}x\int dt\int d^{n}x'\int dt'J(x)G^{0}(x-x')J(x')}\\&\times e^{-\frac{1}{2\hbar^{2}}\int d^{n}x\int dt\int d^{n}x'\int dt'\int_{0}^{\infty}d\omega J_{\omega}(x)G_{\omega}^{0}(x-x')J_{\omega}(x')}.\end{aligned} \qquad (2\text{-}1)$$

Where

$$\begin{aligned}G^{0}(x-x')&=i\hbar\int\frac{d^{n}k\,dk_{0}}{(2\pi)^{n+1}}\frac{e^{-i(k-k_{0})(x-x')}}{k_{0}^{2}-k^{2}+i\varepsilon},\\G_{\omega}^{0}(x-x')&=i\hbar\delta^{n}(x-x')\int\frac{dk_{0}}{2\pi}\frac{e^{ik_{0}(x-x')}}{k_{0}^{2}-\omega^{2}+i\varepsilon}.\end{aligned} \qquad (3\text{-}1)$$

The two-point function can be obtained as

$$G(x-x')=\left(\frac{\hbar}{i}\right)^{2}\frac{\delta^{2}}{\delta J(x)\delta J(x')}W[J,J_{\omega}]\Big|_{j,j_{\omega=0}} \qquad (4\text{-}1)$$

Using Eq.(1-2), we find the following expansion of Green's function in frequency variable

$$G(x-x',\omega)=G^0(x-x',\omega)+ \int_\Omega d^n z_1 G^0(x-z_1,\omega)[\omega^2\tilde{\chi}(\omega,z_1)]G^0(z_1-x',\omega)+ \int_\Omega d^n z_1\int_\Omega d^n z_2 G^0(x-z_1,\omega)[\omega^2\tilde{\chi}(\omega,z_1)]G^0(z_1-z_2,\omega)[\omega^2\tilde{\chi}(\omega,z_2)] G^0(z_2-x',\omega)+... \tag{5-1}$$

Where free energy of the system given by

$$E=-k_BT\sum_{l=0}^{\infty} tr\ln[G(i\upsilon_l;x,x')] \tag{6-1}$$

B. Electromagnetic field

The casimir energy of the medium in the presence of electromagnetic is obtained from two-point Green's function. For this purpose, the total Lagrangian density can be written in coulomb gauge $\nabla.A(x)=0$,as follows

$$L=\frac{1}{2}(E^2-B^2)+\frac{1}{2}\int_0^\infty d\omega(\dot{Y}_\omega^2(x)-\omega^2Y_\omega^2(x))+\int_0^\infty d\omega f(x,\omega)A(x).Y_\omega(x) \tag{1-2}$$

The interacting generating functional is given by

$$W=\int D[A]D[Y_\omega]\exp[\frac{i}{\hbar}\int d^4x(-\frac{1}{2}A_i\hat{K}_{ij}A_j-\frac{1}{2}\int_0^\infty d\omega Y_{i\omega}(\partial_t^2+\omega^2)\delta_{ij}Y_{j\omega} +\int_0^\infty d\omega f(x,\omega)A_i\dot{Y}_{i\omega}+J_iA_i+\int_0^\infty\left(d\omega J_{i\omega}Y_{i\omega}\right))], \tag{2-2}$$

Where $K_{ij}=\left(\frac{\partial_0^2}{c^2}-\nabla^2\right)\delta_{ij}-\partial_i\partial_j$. Using the well-known relation and following the same process that we did for the scalar case, we obtain the following expansion for Green's function

$$G(x-x',\omega)=G^0(x-x',\omega)+ \int_\Omega d^n z_1 G^0(x-z_1,\omega)[\omega^2\tilde{\chi}(\omega,z_1)]G^0(z_1-x',\omega)+ \int_\Omega d^n z_1\int_\Omega d^n z_2 G^0(x-z_1,\omega)[\omega^2\tilde{\chi}(\omega,z_1)]G^0(z_1-z_2,\omega)[\omega^2\tilde{\chi}(\omega,z_2)] G^0(z_2-x',\omega)+... \tag{3-2}$$

Reference

[1] S. Weinberg, *The cosmological constant problem*. Rev. Med. Phys. 61 (1989) 1.

[2] K. A. Milton, G. Romain, I. Gert-Ludwig, L. Astrid, R. Serge, *Negative Casimir entropies in nanoparticle interactions*. Journal of Physics: Condensed Matter. 27 (2015) 21.

[3] U. Stefan, H. Michael, I. Gert-Ludwig, A. Paulo, N. Maia, *Disentangling geometric and dissipative origins of negative Casimir entropies*, arXiv:1507.05891v1

[4] H. B. G. Casimir, *On the attraction between two perfectly conducting plates*. Proc. K. Ned. Akad. Wet. 51 (1948) 793.

[5] I. E. Dzyaloshinskii, E. M. Lifshitz, L. P. Pitaevskii, *General theory of vander waals' forses*, sov. Phys. Usp. 4 (1961) 153.

[6] K. A. Milton, E. K. Abalo, P. Parashar, N. Pourtolami, I. Brevik, S. A. Ellingsen, S. Y. Buhmann, S. Scheel, *Casimir-Polder repulsion: Three-body effects*. Phys. Rev. A .91 (2015) 042510.

[7] P. L. Pitaevskii, *Casimir-lifshtze forces and entropy*, Int. J. Mod. Phys. A. 25 (2010) 2313.

[8] T. Emig, N. Graham, R. L. Jaffre, M. Kardar, *Casimir Forces between Arbitrary Compact Objects*. Phys. Rev. Lett. 99, (2007) 170403.

[9] S. J. Rahi, T. Emig, N. Graham, R. L. Jaffre, M. Kardar, *Scattering theory approach to electrodynamic Casimir forces* . Phys. Rev. D 80 (2009) 085021.

[10] A. Lambrecht, P. A. Maia Neto, S. Reynaud, *The Casimir effect within scattering theory* .New Journal of physics 8 (2006)243.

[11] K. A. Milton, J. Wagner, Exact *Casimir Interaction Between Semitransparent Spheres and Cylinders*. Phys. Rev. D 77, (2008) 045005.

[12] K. Milton. P. Parashar and J. Wagner, *Exact results for Casimir interactions between dielectric bodies: The weak-coupling or van der Waals limit*, Phys. Rev. Lett. 101 (2008) 160402.

[13] G. Ingold, S. Umrath, M. Hartmann, R. Guerout, A. Lambrecht, S. Reynaud, and K. Milton, *Geometric origin of negative Casimir entropies: A scattering channel analysis*, Phys. Rev. E 91 (2015) 033203 .

[14] F. Kheirandish, M. Jafari, *perturbative approach to calculating the casimir force in fluctuating scalar and vector fields*. Phys. Rew A. 86 (2012) 022503.

[15] V. B. Bezerra, G. L. Klimchitskaya, V. M. Mostepanenko, *Thermodynamical aspects of the Casimir force between real metals at nonzero temperature*. Phys. Rev. A. 65 (2002) 052113.

[16] M. Bordag, I. G. Pirozhenko, *Casimir entropy for a ball in front of a plane* . Phys. Rev. D. 82 (2010) 125016.

[17] P. Rodriguez-Lopez, *Casimir energy and entropy in the sphere-sphere geometry*. Phys. Rev. B. 84 (2011) 075431.

[18] B. Geyer, *On thermal Casimir force between real metals*. Journal of Physics: Conference Series, 1 (2009)
[19] G. L. Ingold, P. Hanggi, P. Talker, *Specific heat anomalies of open quantum systems*. Phys.Rev. E. 79 (2009) 061105.
[20] F. Kheirandish, S. Salimi., *Quantum field theory in the presence of a medium Green's function expansions*. Phys. Rev. A. 84 (2011) 062122.
[21] W. Greiner, J. Richardt, *Field quantization*. Springer-Verlag, Berlin, Heidelberg (1996).
[22] J. I. Kapusta, *Finite-Temperature Field Theory*. Cambridge University Press, Cambridge (1989).